Mark scheme for Support Worksheet – Topic 2, Worksheet 4

1 The time is found from $y = \frac{1}{2}gt^2 \Rightarrow t = \frac{2y}{g}$; i.e. $t = \frac{2 \times 0.80}{9.8} = 0.40$ s [2]

2 $x = vt = 4.6 \times 0.40 = 1.8$ m [1]

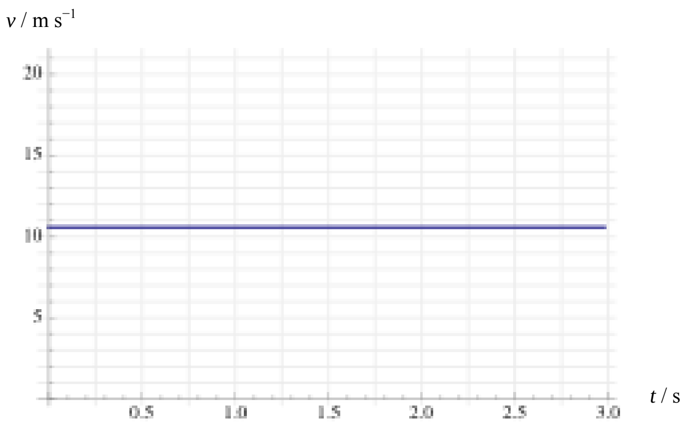
3 **a** The initial vertical component of velocity is
$u_y = u\sin\theta = 18 \times \sin 54^\circ = 14.56\ \mathrm{m\,s^{-1}}$; Then
$v_y^2 = (u\sin\theta)^2 - 2gy \Rightarrow y = \frac{(u\sin\theta)^2}{2g} = \frac{14.56^2}{2 \times 9.8} = 10.8 \approx 11$ m [2]

OR The vertical component of velocity becomes zero when
$0 = u\sin\theta - gt \Rightarrow t = \frac{14.56}{9.8} = 1.49$ s; and so $y = ut\sin\theta - \frac{1}{2}gt^2 = 10.8 \approx 11$ m [2]

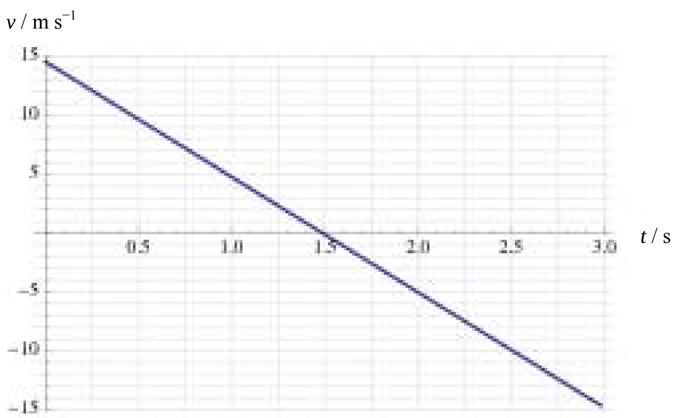
b The ball will land when $t = 2 \times 1.49 = 2.98$ s; and so
$x = ut\cos\theta = 18 \times \sin 54^\circ \times 2.98 = 31.53 \approx 32$ m [2]

(For completeness, the following graphs show the variation with time of the velocity components – the exercise does not require these.)

a



b



4 **a** The initial vertical component of velocity is

$u_y = u\sin\theta = 28 \times \sin 38^{\circ} = 17.24\ \text{m s}^{-1}$; Then

$v_y^2 = (u\sin\theta)^2 - 2gy \Rightarrow y = \dfrac{(u\sin\theta)^2}{2g} = \dfrac{17.24^2}{2\times 9.8} = 15.16 \approx 15\ \text{m}$ is the maximum height from the cliff. [2]

b $v_y = u\sin\theta - gt = 0 \Rightarrow t = \dfrac{17.24}{9.8} = 1.76\ \text{s}$ [1]

c Considering the origin of the axes to now be at the highest point we then have

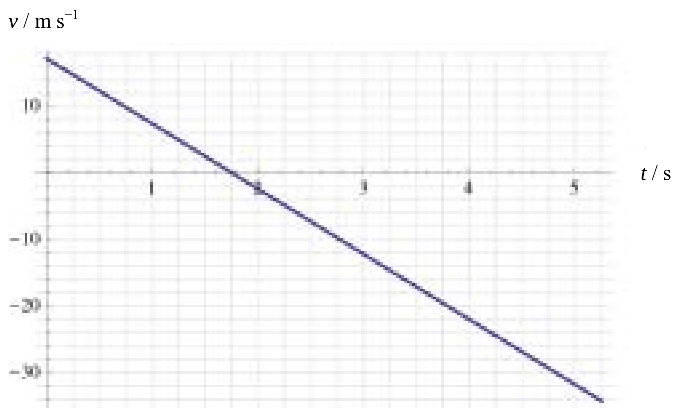
$y = -\dfrac{1}{2}gt^2 \Rightarrow -60.16 = -\dfrac{1}{2}\times 9.8 \times t^2$ and so $t = 3.50\ \text{s}$; Hence the total time to reach the sea from the launch is $t = 1.76 + 3.50 = 5.26\ \text{s}$ and so

$x = ut\cos\theta = 28 \times \sin 38^{\circ} \times 5.26 = 116 \approx 120\ \text{m}$ [2]

5 **a**

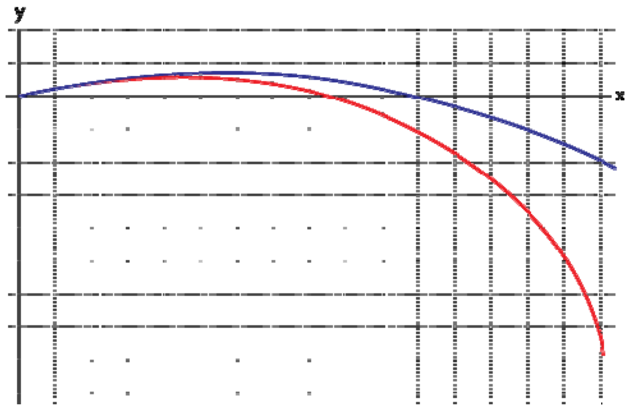
[1]

b



[2]

6



[2]

7 **a** The gravitational potential is $V = -\frac{GM}{r/2} - \frac{GM}{r/2} = -\frac{4GM}{r}$ [1]

b $g = \frac{GM}{\left(\frac{r}{2}\right)^2} - \frac{GM}{\left(\frac{r}{2}\right)^2} = 0$ [1]

8 We have that $g = \frac{G81M}{r_1^2} - \frac{GM}{r_2^2} = 0$; and so $\frac{r_1}{r_2} = 9$ [2]

9 **a** From $v^2 = \frac{GM}{R}$ and $v = \frac{2\pi R}{T}$ we obtain $\left(\frac{2\pi R}{T}\right)^2 = \frac{GM}{R}$ and so $T^2 = \frac{4\pi^2 R^3}{GM}$;

Now remember that $g = \frac{GM}{R^2} \Rightarrow GM = gR^2$; Substituting gives the result

$T = 2\pi\sqrt{\frac{R}{g}}$. [2]

b $T = 2\pi\sqrt{\frac{6.4\times10^6}{9.8}} = 5078\,\text{s} = 85\,\text{min}$ [1]

10 It is not possible since the gravitational force points towards the centre of the Earth; and we require a force to point towards the centre of the circular orbit in order to provide the centripetal force. [2]

11 **a** $\frac{GMm}{r^2} = \frac{mv^2}{2}$; and so by simplifying we get $v = \sqrt{\frac{GM}{r}}$ [2]

b $E_K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$ [1]

c $E_T = \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2r}$ [1]

12 **a** A frictional force will reduce the total energy of the satellite; and so from $E_T = -\frac{GMm}{2r}$ the orbit radius must decrease. [2]

b From $v = \sqrt{\frac{GM}{r}}$ if r decreases the speed will increase. [1]

c The extra energy comes from the decrease in gravitational potential energy. [1]

